# Reformulating Constraint Satisfaction Problems with Application to Geospatial Reasoning

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# Contributions

- BID problem as a CSP [Michalowski & Knoblock, AAAI 05]
  – Improved constraint model
  – Showed original BID problem is in **P**
  – Custom solver

- Four new reformulation techniques for CSPs
  1. Query reformulation
  2. Domain reformulation
  3. Constraint relaxation
  4. Reformulation via symmetry detection

- Applying the reformulations to the BID problem

# Outline

- **Background**
- BID:  CSP model & custom solver
- Reformulation techniques
  - Description
  - General use in CSPs
  - Application to BID
  - Evaluation on real-world BID data
- Conclusions & future work

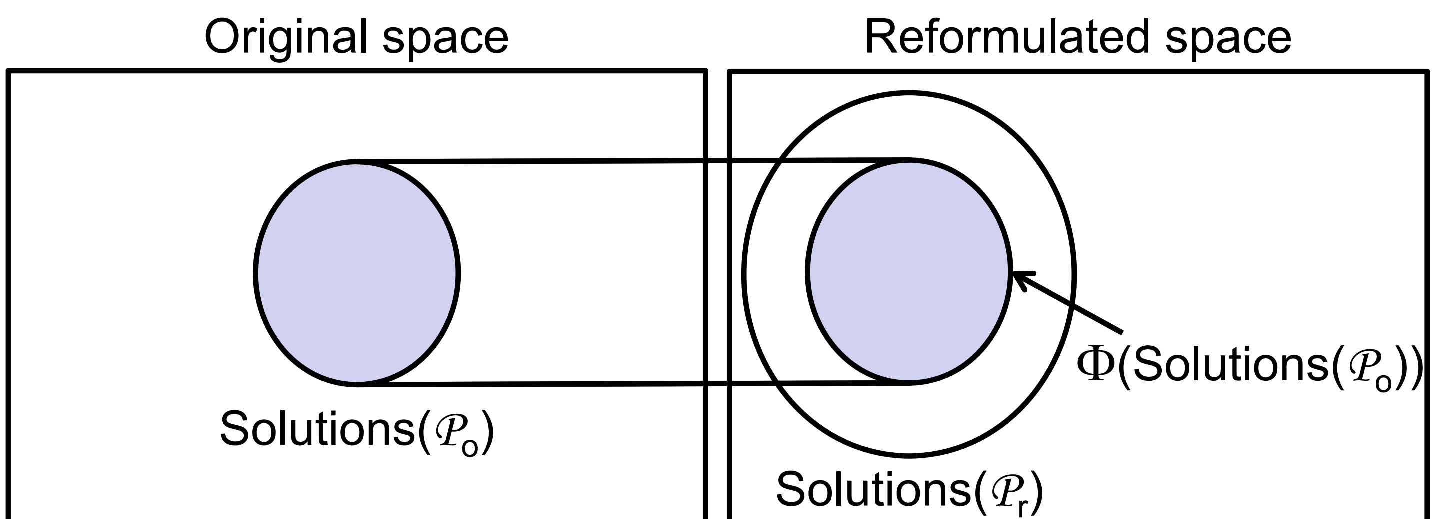
# Abstraction & Reformulation

Original problem
- Original formulation
- Original query

*Reformulation technique* →

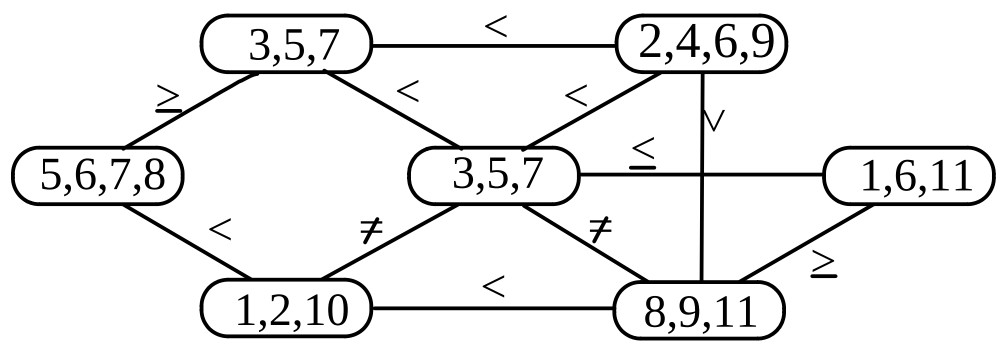
Reformulated problem
- Reformulated formulation
- Reformulated query

… may be an approximation



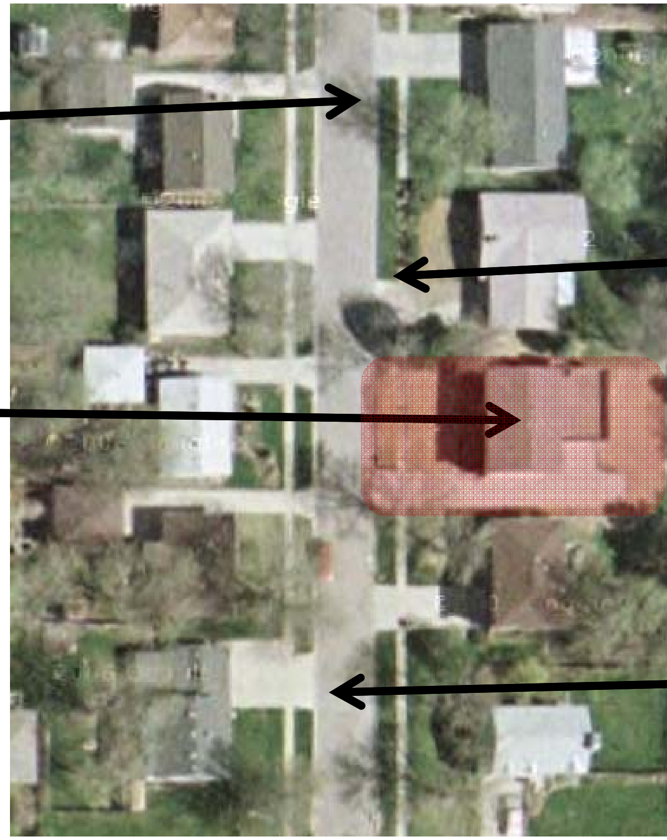
# Constraint Satisfaction Problems

- Formulation: $\mathcal{F} = (\mathcal{V}, \mathcal{D}, \mathcal{C})$
  - $\mathcal{V}$ = set of variables
  - $\mathcal{D}$ = set of their domains
  - $\mathcal{C}$ = set of constraints restricting the acceptable combination of values for variables
- Query: All solutions, a single solution, *etc.*
- Solved with
  - Constraint propagation
  - Search



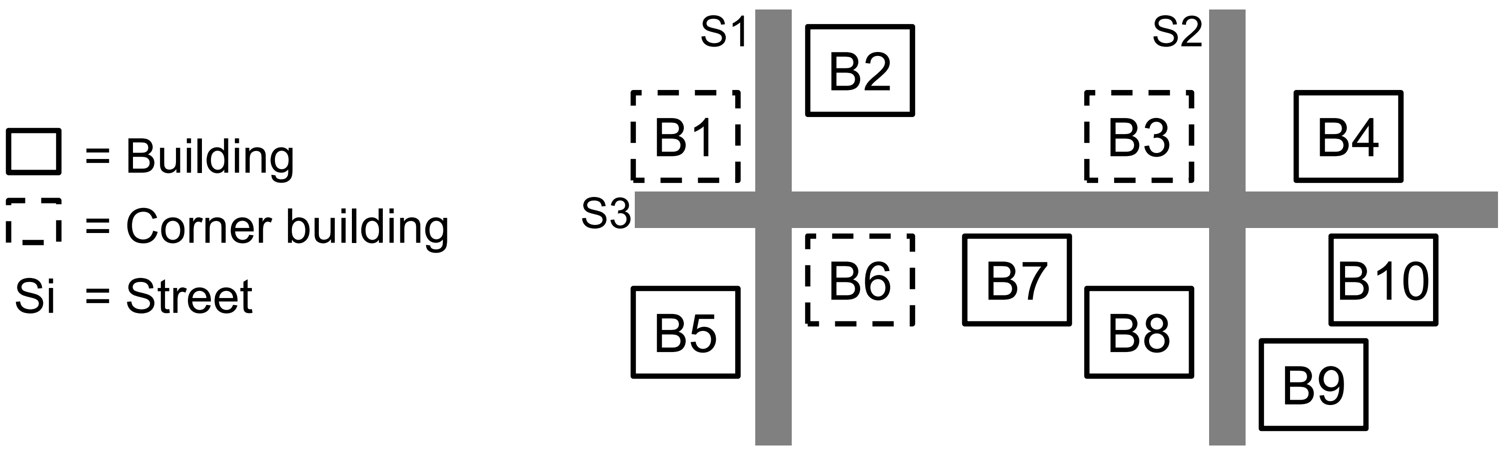

- Term: variable-value pair (vvp)

# Issue: finding Ken’s house



# Building Identification (BID) problem

- Layout: streets and buildings

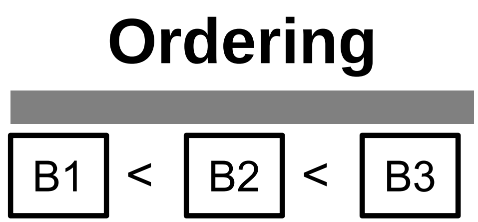


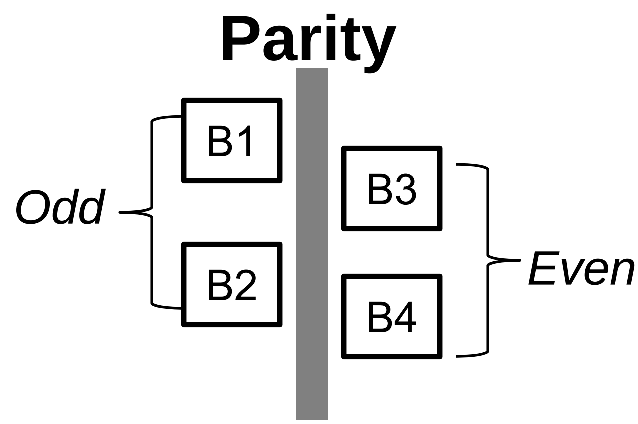

- Phone book
  - Complete/incomplete
  - Assumption: all addresses in phone book must be used

S1#1, S1#4, S1#8,
S2#7, S2#8, S3#1,
S3#2, S3#3, S3#15,
…

# Basic (address numbering) rules

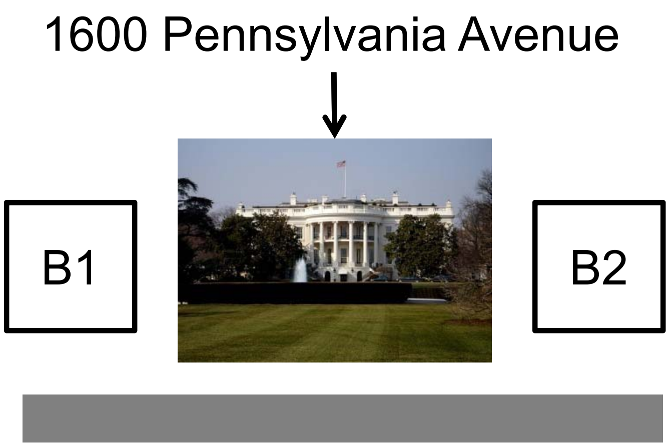
- Ordering
  - Numbers increase/decrease along a street
- Parity
  - Numbers on a given side of a street are odd/even



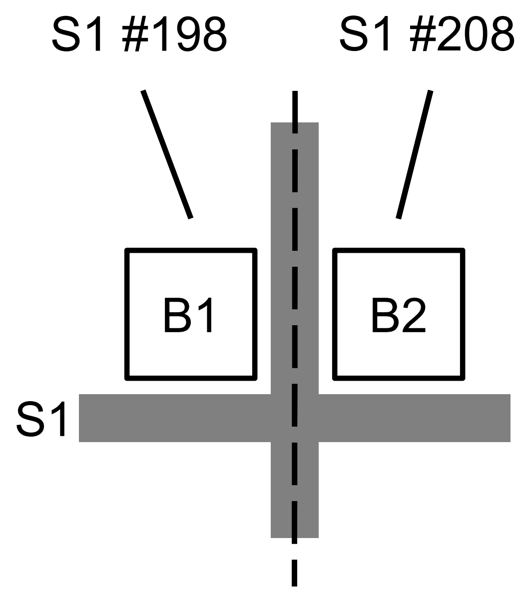



# Additional information

## Landmarks

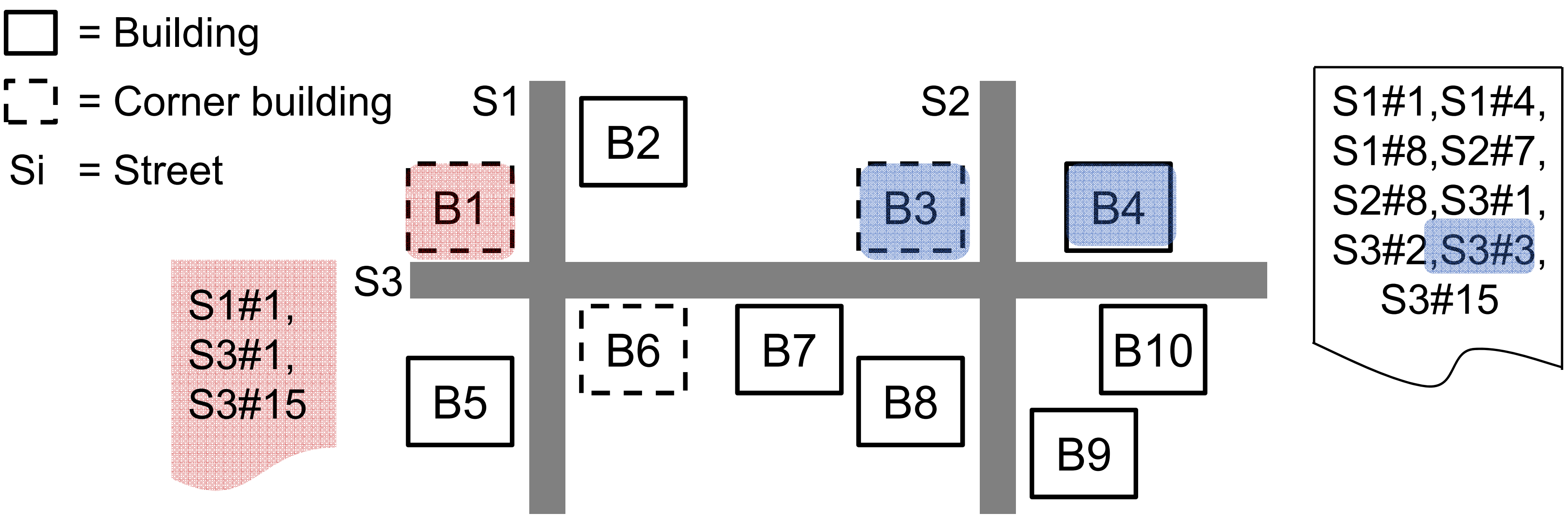



## Gridlines



# Query

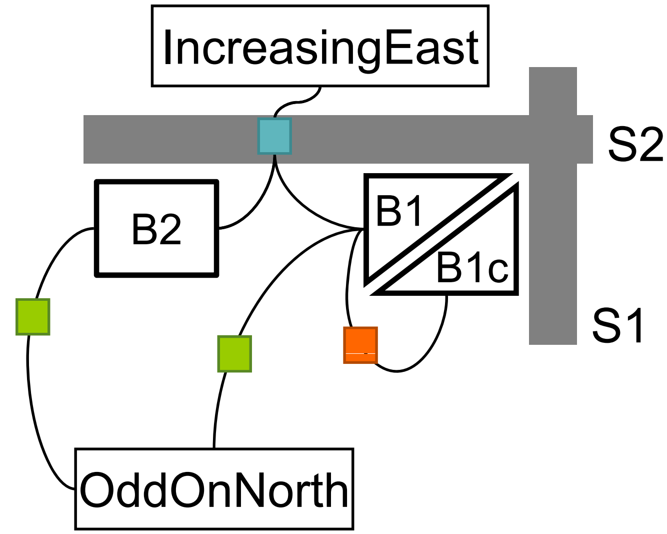
1. Given an address, what buildings could it be?
2. Given a building, what addresses could it have?



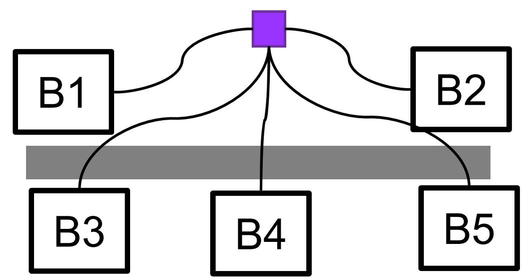
# Outline

- Background
- **BID model & custom solver**
- Reformulation techniques
- Conclusions & future work

# CSP model

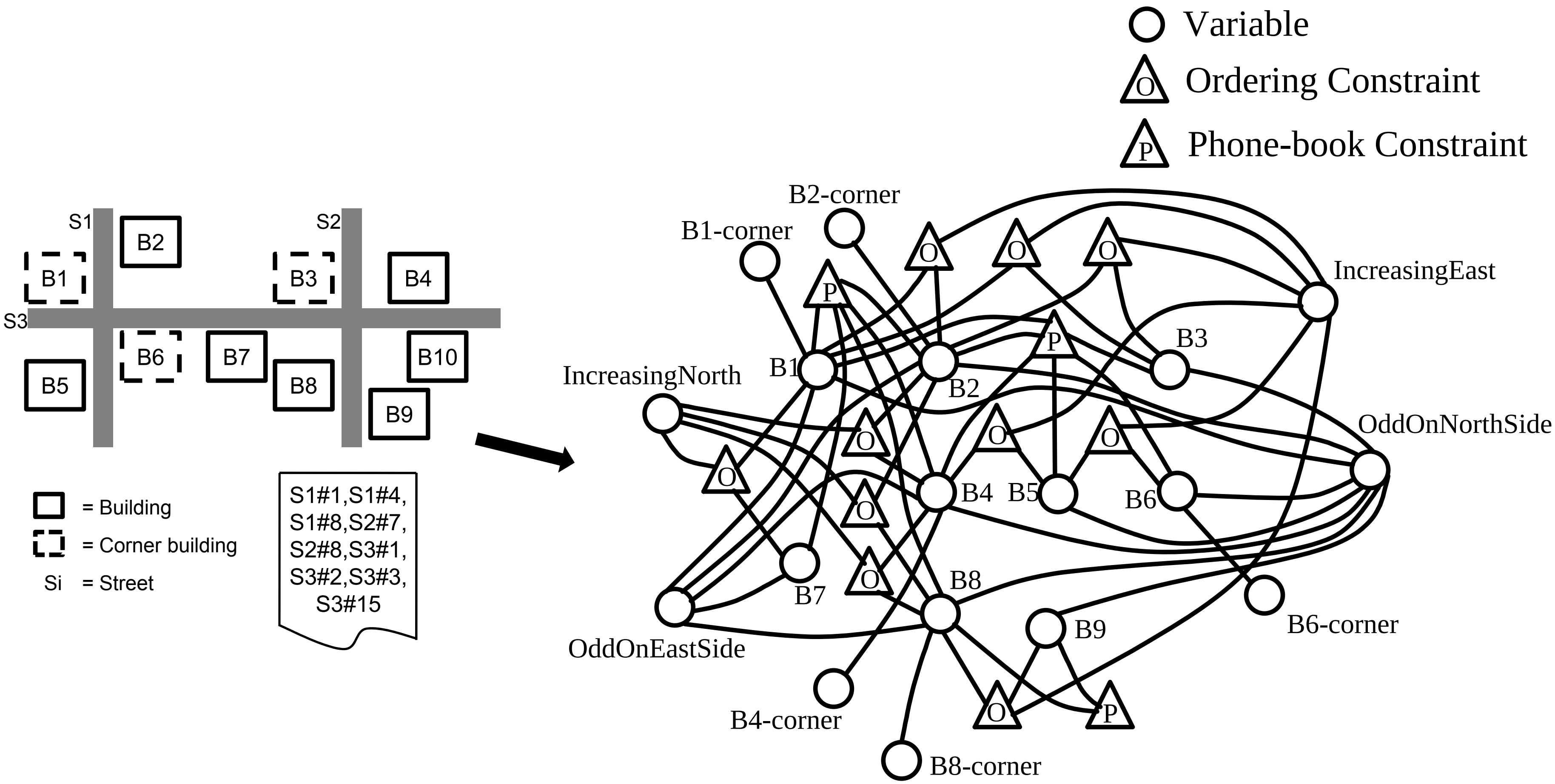



- Parity constraints
- Ordering constraints
- Corner constraints




- Phone-book constraints
- Optional: grid constraints

# Example constraint network



# Features of new model & solver

- Improvement over previous work [Michalowski +, 05]

- Model
  - Reflects topology
  - Reduces number of variables and constraint arity
  - Constraints can be declared locally & in restricted 'contexts' (feature important for Michalowski's work)

- Solver
  - Exploits structure of problem (backdoor variables)
  - Implements domains as (possibly infinite) intervals
  - *Incorporates all reformulations (to be introduced)*

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  - **Query reformulation**
  - AllDiff-Atmost & domain reformulation
  - Constraint relaxation
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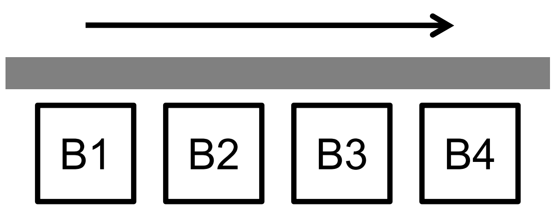
# Query in the BID

- Problem:  BID instances have many solutions

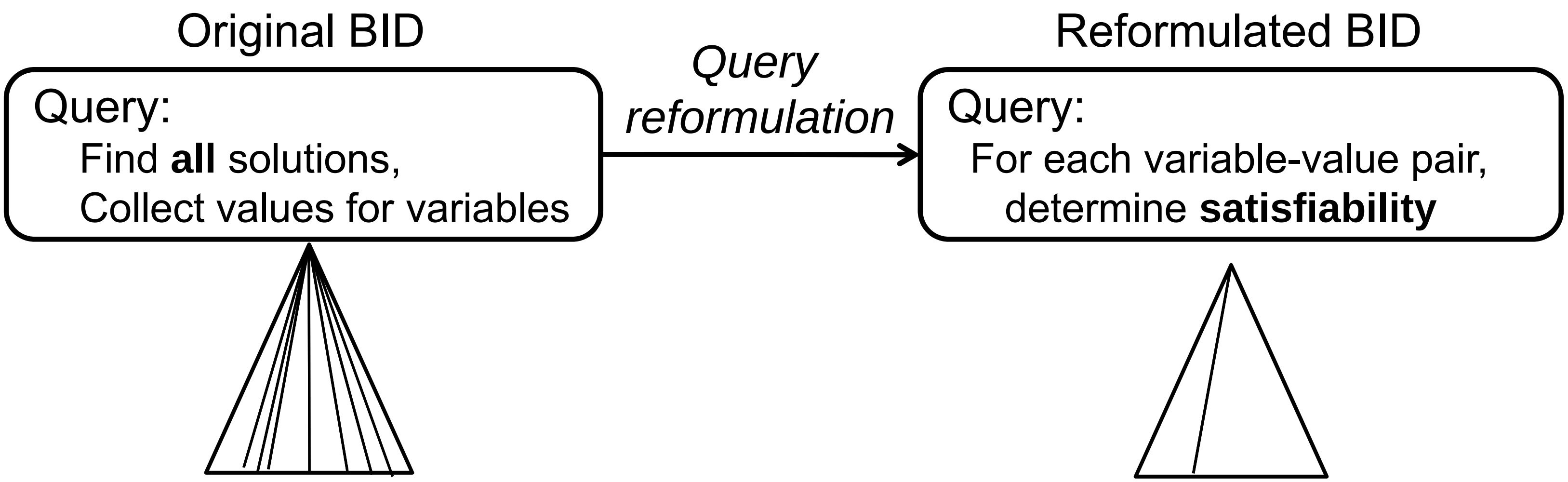



Phone book: {4,8}

| B1 | B2 | B3 | B4 |
|---|---|---|---|
| 2 | 4 | 6 | 8 |
| 2 | 4 | 8 | 10 |
| 2 | 4 | 8 | 12 |
| 4 | 8 | 10 | 12 |
| 4 | 6 | 8 | 10 |
| 4 | 6 | 8 | 12 |

We **only** need to know which values (address) appear in ***at least one*** solution for a variable (building)

# Query reformulation




| Original query | Reformulated query |
|---|---|
| Single counting problem | Many satisfiability problems |
| All solutions | Per-variable solution |
| Exhaustive search | One path |
| Impractical when there are many solutions | Costly when there are few solutions |

# **Evaluations:** real-world data from El Segundo

[Shewale]

| Case study | Phone book | Number of… | | |
|---|---|---|---|---|
| | Completeness | Buildings | Corner buildings | Blocks |
| NSeg125-c<br>NSeg125-i | 100.0%<br>45.6% | 125 | 17 | 4 |
| NSeg206-c<br>NSeg206-I | 100.0%<br>50.5% | 206 | 28 | 7 |
| SSeg131-c<br>SSeg131-i | 100.0%<br>60.3% | 131 | 36 | 8 |
| SSeg178-c<br>SSeg178-i | 100.0%<br>65.6% | 178 | 46 | 12 |

Previous work did not scale up beyond 34 bldgs, 7 corner bldgs, 1 block

# Evaluation: query reformulation

Incomplete phone book → many solutions → better performance

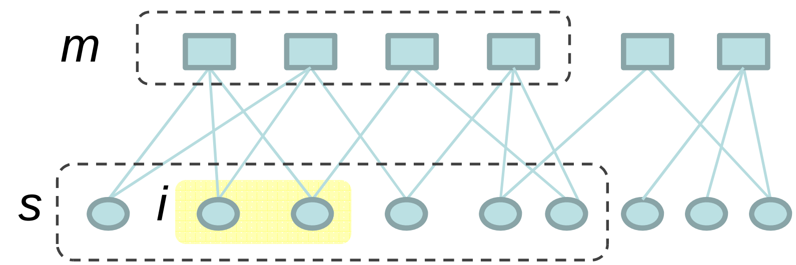
| Case study | Original query | New query [s] |
|---|---|---|
| NSeg125-i | >1 week | 744.7 |
| NSeg206-i | >1 week | 14,818.9 |
| SSeg131-i | >1 week | 66,901.1 |
| SSeg178-i | >1 week | 119,002.4 |

Complete phone book → few solutions → worse performance

| Case study | Original query [s] | New query [s] |
|---|---|---|
| NSeg125-c | 1.5 | 139.2 |
| NSeg206-c | 20.2 | 4,971.2 |
| SSeg131-c | 1123.4 | 38,618.4 |
| SSeg178-c | 3291.2 | 117,279.1 |

# Generalizing query reformulation

- Relational ($i$,$m$)-consistency, algorithm R($i$,$m$)C
  - For every $m$ constraints
    - Compute **all solutions** of length $s$
    - To generate tuples of length $i$
  - Space: $O(d^{s})$




- Reformulated BID query is R(1,$|\mathcal{C}|$)C

- Query reformulation for Relational ($i$,$m$)-consistency
  - For each combination of values for $i$ variables
    - Try to extend to **one** solution of length $s$
  - Space: $O(\binom{s}{i} d^{i})$, $i < s$

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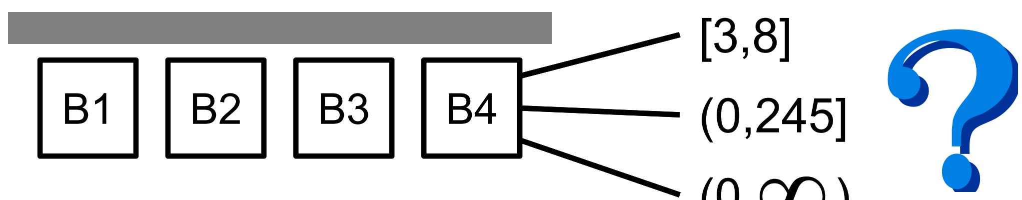
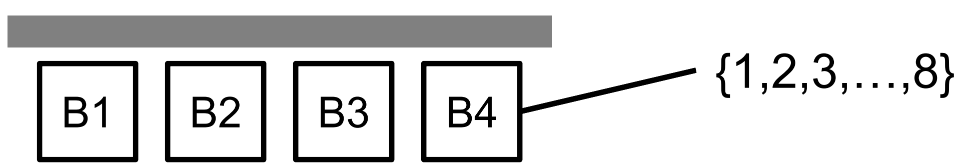
# Domain reformulation

- Domains in the BID are large
- Min/max value?



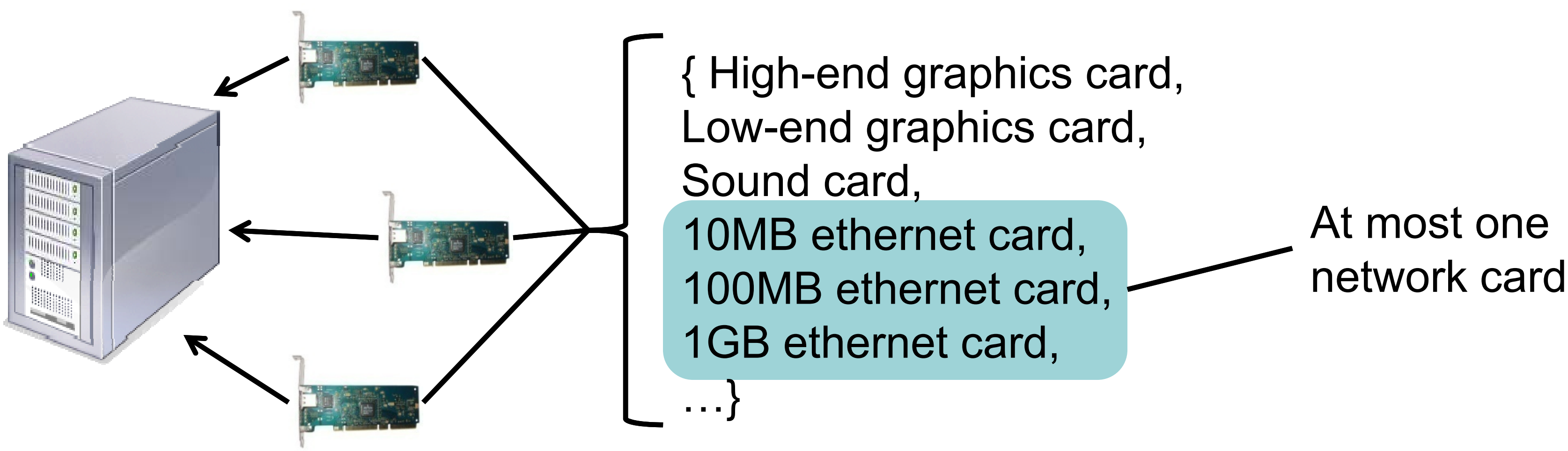

Phonebook = {3,8}

- Enumerate?



# AllDiff-Atmost constraint

- AllDiff-Atmost($\mathcal{A}$,*k*,*d*)
  - The variables in $\mathcal{A}$ can be assigned at most *k* values from the set *d*



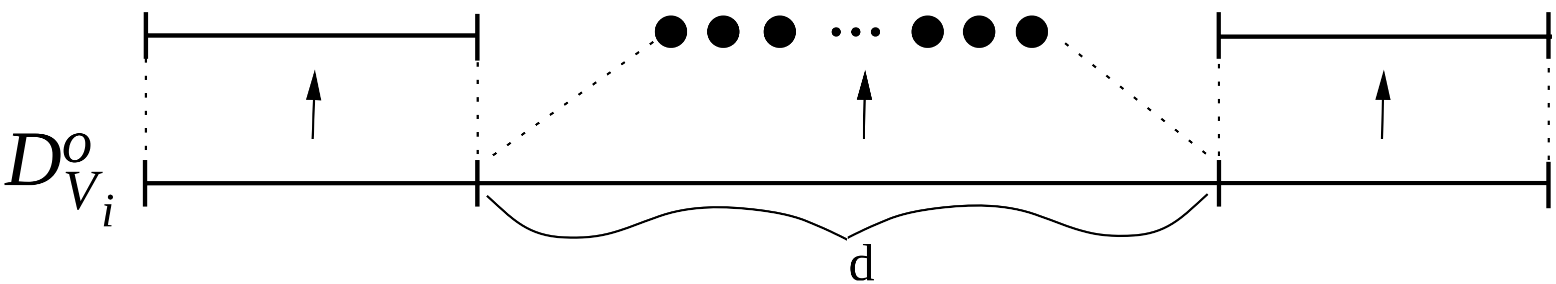

Three expansion slots

# AllDiff-Atmost reformulation

Replaces

- interval *d* of values (potentially infinite)
- with *k* **symbolic values**

$$D^{ref}_{V_i} = D^{ref,l}_{V_i} \cup \{S_1, S_2, \ldots, S_k\} \cup D^{ref,r}_{V_i}$$



# AllDiff-Atmost in the BID

Even side B1 B2 B3 B4 B5

Phone book: {12,28}

Original domain = {2, 4, …, 998, 1000}

| | | | | |
|---|---|---|---|---|
| **12** | **28** | 30 | 32 | 34 |
| **12** | 14 | 16 | **28** | 36 |
| 10 | **12** | 14 | 20 | **28** |
| 2 | 4 | 6 | **12** | **28** |
| … | … | **12** | **28** | **…** |

- Can use at most
  - **3** addresses in [2,12)
  - **3** addresses in (12,28)
  - **3** addresses in (28,1000]

Reformulated domain { $s_1$, $s_2$, $s_3$, 12, $s_4$, $s_5$, $s_6$, 28, $s_7$, $s_8$, $s_9$ }

Original domain { 2, 4, …, 10, 12, 14, …, 26, 28, 30, …, 998, 1000 }

# Evaluation: domain reformulation

- Reduced domain size → improved search performance

| Case study | Phone-book completeness | Average domain size | | Runtime [s] | |
|---|---|---|---|---|---|
| | | Original | Reformulated | Original | Reformulated |
| NSeg125-i | 45.6% | 1103.1 | 236.1 | 2943.7 | 744.7 |
| NSeg206-i | 50.5% | 1102.0 | 438.8 | 14,818.9 | 5533.8 |
| SSeg131-i | 60.3% | 792.9 | 192.9 | 67,910.1 | 66,901.1 |
| SSeg178-i | 65.6% | 785.5 | 186.3 | 119,002.4 | 117,826.7 |

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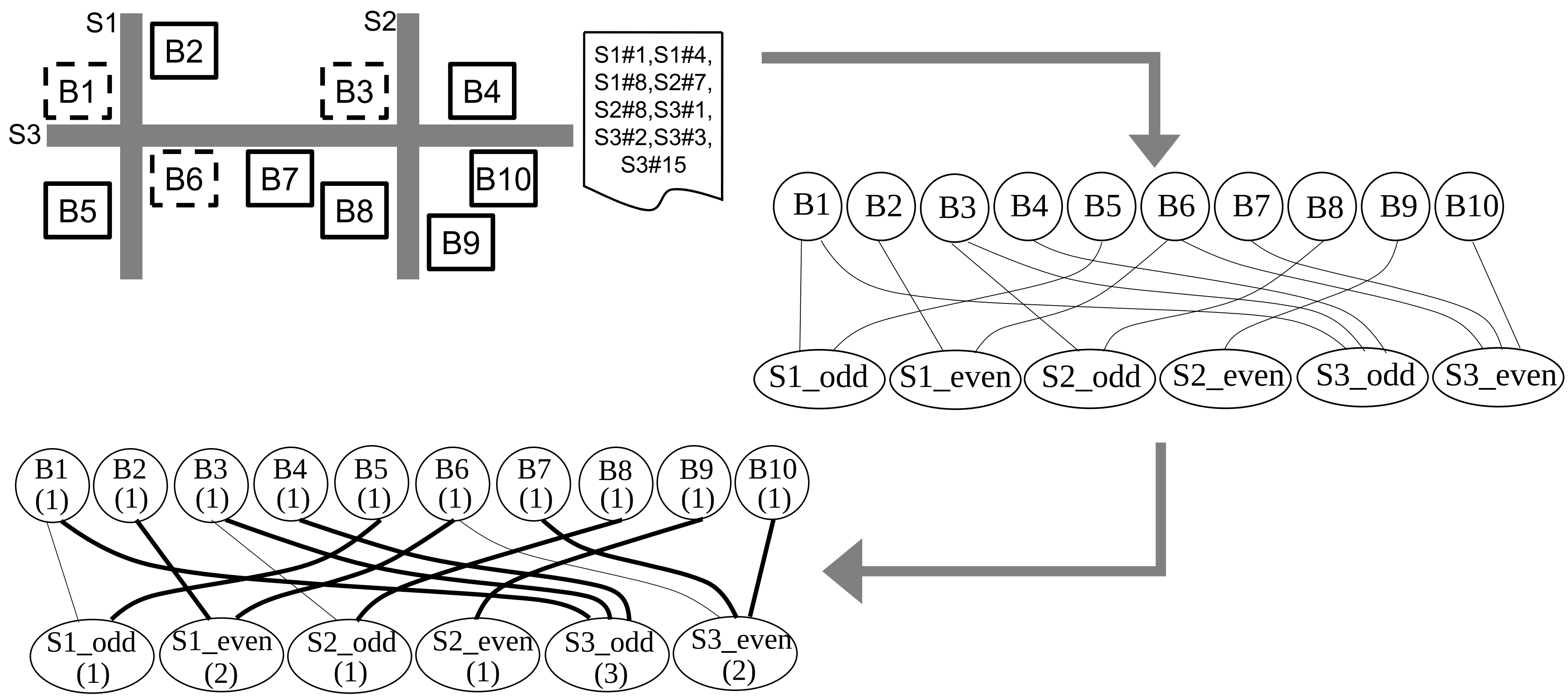
# BID as a matching problem

- Assume we have no grid constraints



# BID w/o grid constraints

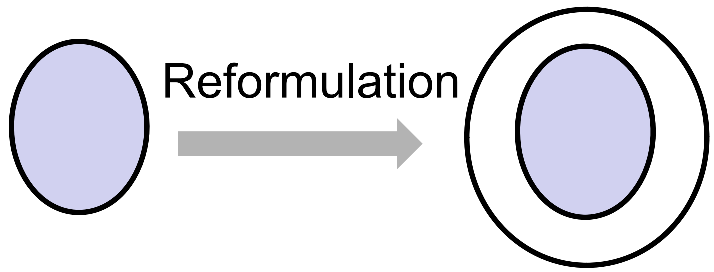
- BID instances without grid constraints can be solved in ***polynomial time***

| Case study | Runtime [s] | |
|---|---|---|
| | BT search | Matching |
| NSeg125-c | 139.2 | 4.8 |
| NSeg206-c | 4971.2 | 16.3 |
| SSeg131-c | 38618.3 | 7.3 |
| SSeg178-c | 117279.1 | 22.5 |
| NSeg125-i | 744.7 | 2.5 |
| NSeg206-i | 5533.8 | 8.5 |
| SSeg131-i | 38618.3 | 7.3 |
| SSeg178-i | 117826.7 | 4.9 |

# BID w/ grid constraints

1. Matching reformulation as a *necessary approximation* of the BID

Solutions to BID instance




Solutions to the matching reformulation

**No solution to the original BID** ⟵ **No solution to matching reformulation**

2. Filtering

[Régin, 1994]

**Remove vvps that do not appear in a maximum matching**

# Matching reformulation in Solver

- **Remove vvps that do not… Preproc1**
- For every vvp
  - Consider CSP + vvp
  - **Is the relaxed CSP solvable? Preproc2**
    - Find one solution using BT search
      - **After each variable instantiation, remove vvps that do …. Lookahead**

# Evaluation: matching reformulation

- Generally, improves performance

| Case Study | BT | Preproc2 +BT | % (from BT) | Lkhd +BT | % (from BT) | Lkhd +Preproc1&2 + BT | % (from Lkhd+BT) |
|---|---|---|---|---|---|---|---|
| NSeg125-i | 1232.5 | 1159.1 | **6.0%** | 726.6 | **41.0%** | 701.1 | **3.5%** |
| NSeg206-c | 2277.5 | 614.2 | **73.0%** | 1559.2 | **31.5%** | 443.8 | **71.5%** |
| SSeg178-i | 138404.2 | 103244.7 | **25.4%** | 121492.4 | **12.2%** | 85185.9 | **29.9%** |

- Rarely, the overhead exceeds the gains

| Case Study | BT | Preproc2 +BT | % (from BT) | Lkhd +BT | % (from BT) | Lkhd +Preproc1&2 + BT | % (from Lkhd+BT) |
|---|---|---|---|---|---|---|---|
| NSeg125-c | 100.8 | 33.2 | **67.1%** | **140.2** | **-39.0%** | 29.8 | **78.7%** |
| NSeg131-i | 114405.9 | 114141.3 | **0.2%** | 107896.3 | **5.7%** | **108646.6** | **-0.7%** |

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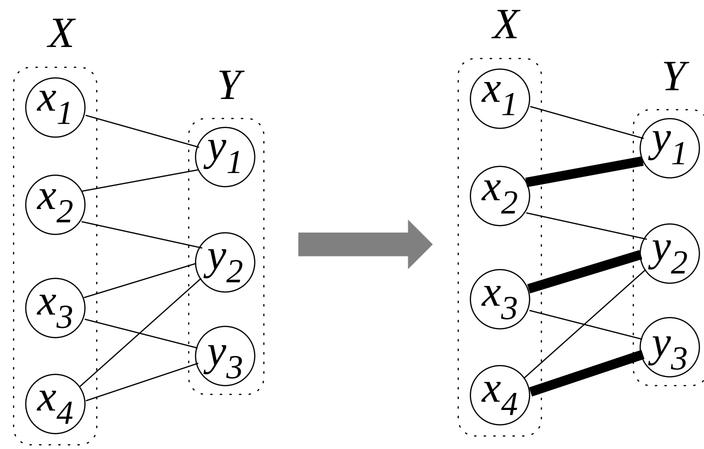
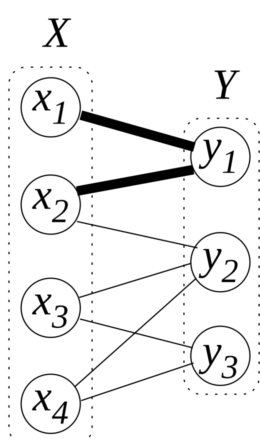
# Find all maximum matchings

Find one maximum matching [Hopcroft+Karp, 73]

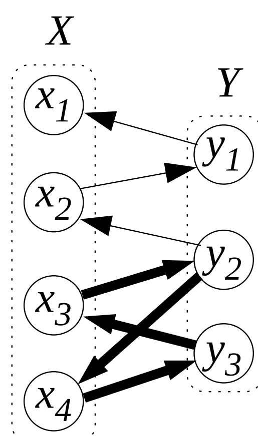



Identify… [Berge, 73]

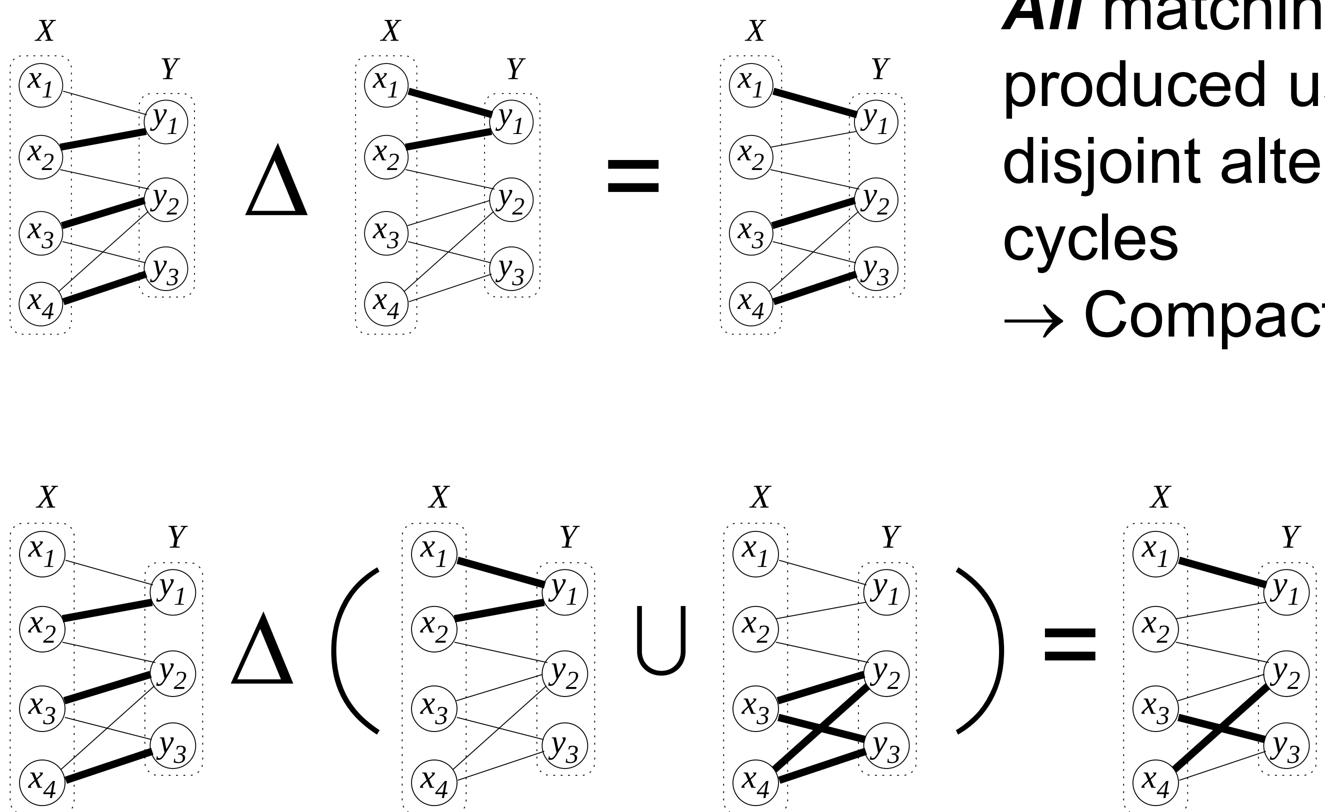
… even alternating paths starting @ a free vertex:




… alternating cycles:

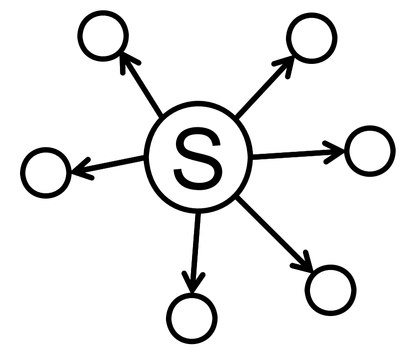


# Symmetric maximum matchings

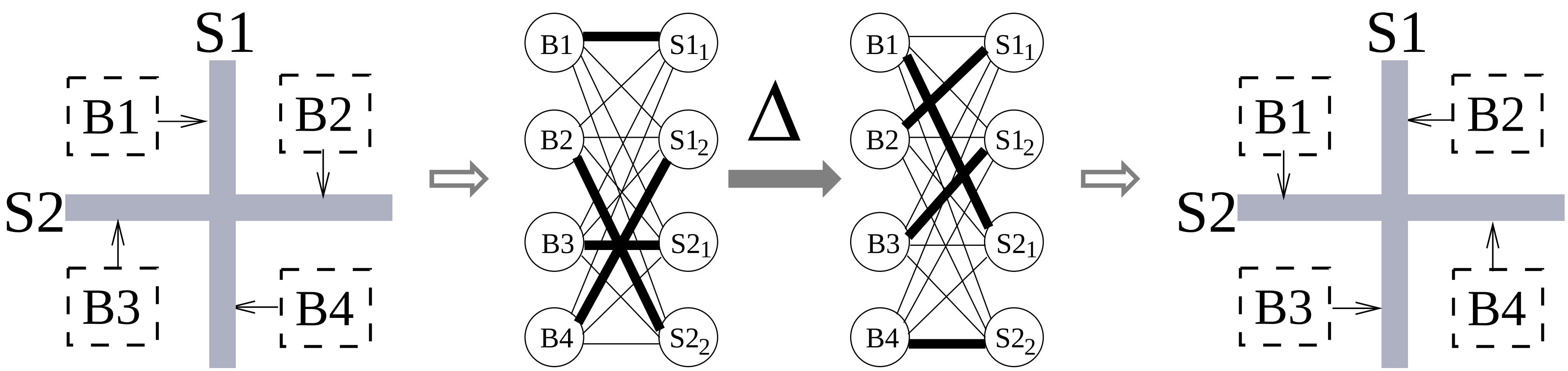



***All*** matchings can be produced using the sets of disjoint alternating paths & cycles

→ Compact representation



# Symmetric matchings in BID




- Some symmetric solutions do not break the grid constraints
  - Avoid exploring symmetric solutions during search
- Some do, we do not know how to use them…

# Conclusions

- We proposed four reformulation techniques
- We described their usefulness for general CSPs
- We demonstrated their effectiveness on the BID
- **Lesson: reformulation is an effective approach to improve the scalability of complex systems**

# Future work

- Empirically evaluate our new algorithm for relational (*i*,*m*)*-consistency*
- Exploit the symmetries we identified
- Enhance the model by incorporating new constraints [Michalowski]

# Questions?