Reformulating CSPs for Scalability with Application to Geospatial Reasoning

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Supported by NSF CAREER Award #0133568 and
AFOSR grants FA9550-04-1-0105 and FA9550-07-1-0416

Contributions

- BID problem as a CSP [Michalowski & Knoblock, AAAI 05]
  - Improved constraint model
  - Showed original BID problem is in **P**
  - Custom solver

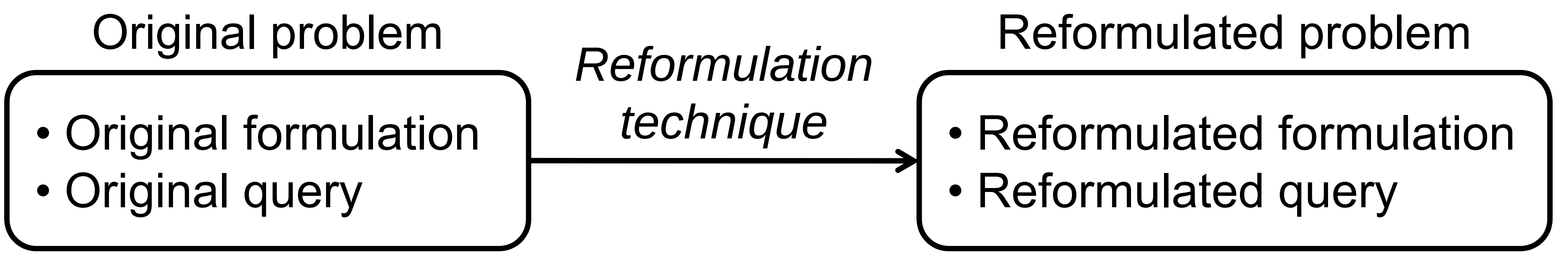
- Four new reformulation techniques for CSPs
  1. Query reformulation
  2. Domain reformulation
  3. Constraint relaxation
  4. Reformulation via symmetry detection

- Applying the reformulations to the BID problem

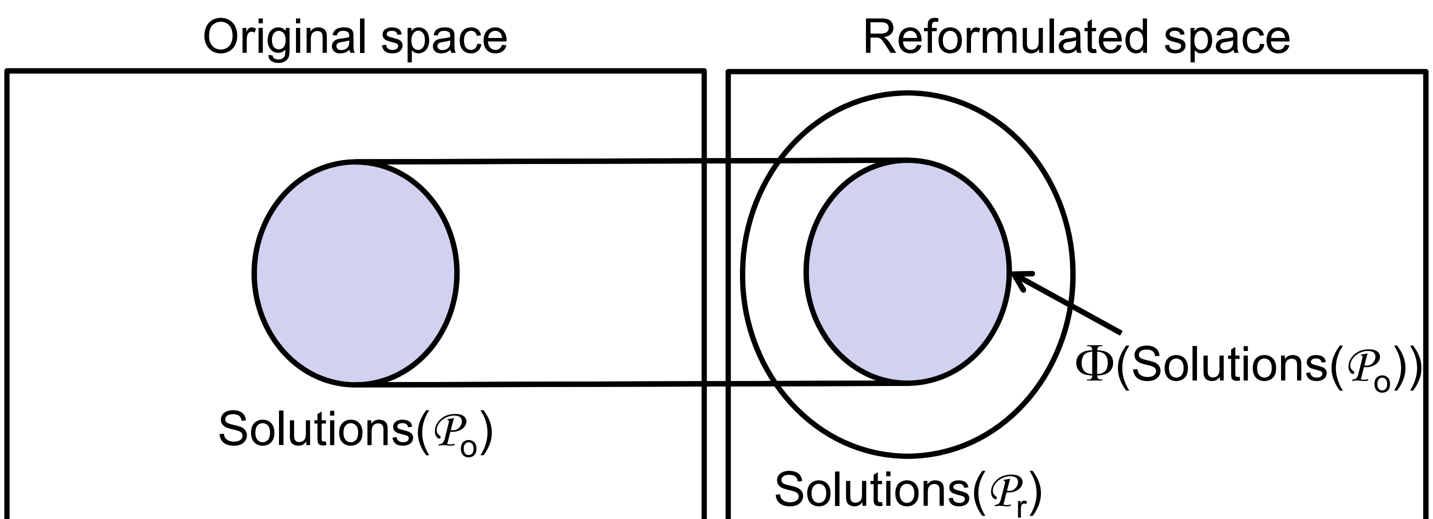
Outline

- **Background**
- BID: CSP model & custom solver
- Reformulation techniques
  - Description
  - General use in CSPs
  - Application to BID
  - Evaluation on real-world BID data
- Conclusions & future work

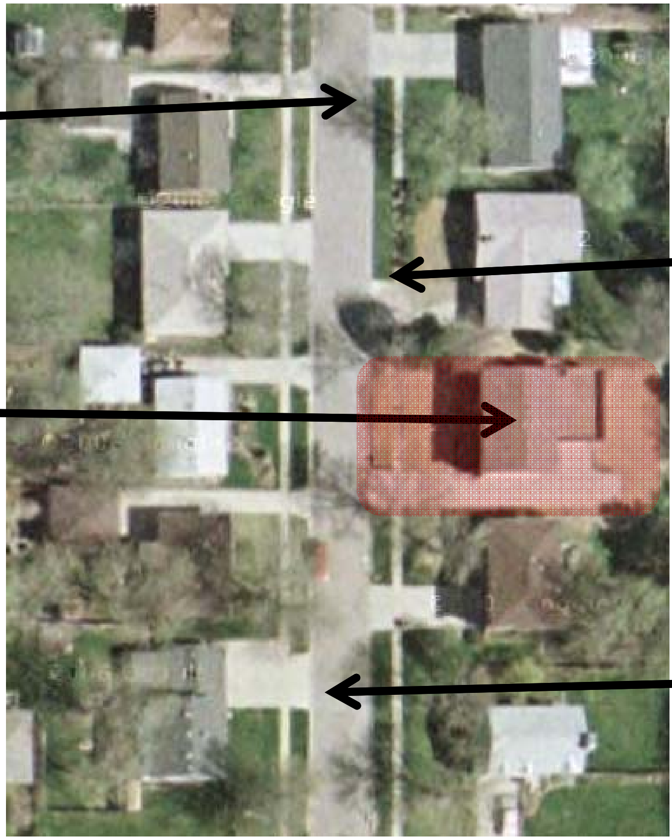
Abstraction & Reformulation



The reformulation may be an approximation

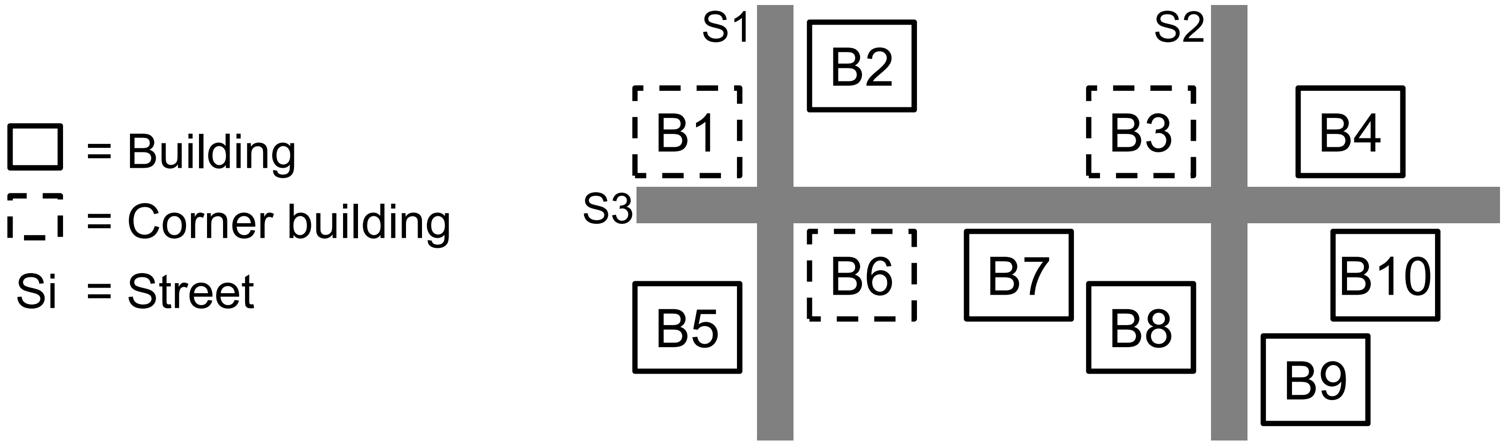


Issue: finding Ken's house



Building Identification (BID) problem

- Layout: streets and buildings

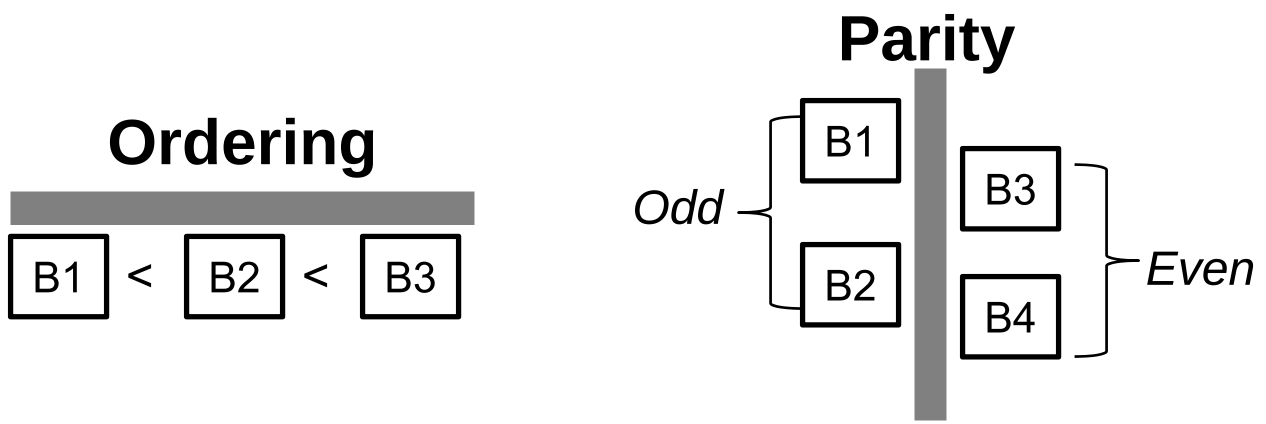


- Phone book
  - Complete/incomplete
  - Assumption: all addresses in phone book correspond to a building in the layout

S1#1, S1#4, S1#8,
S2#7, S2#8, S3#1,
S3#2, S3#3, S3#15,
…

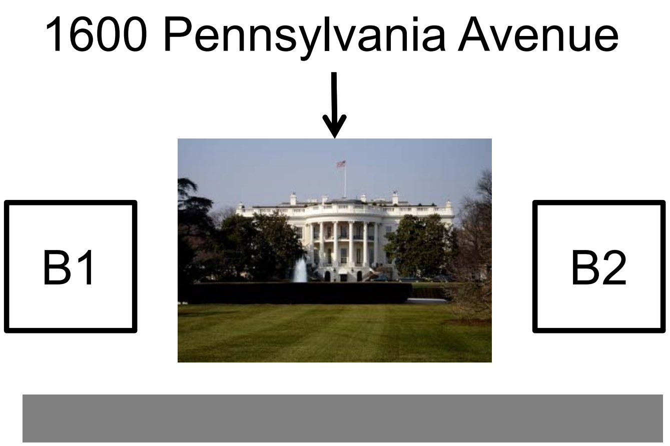
Basic (address numbering) rules

- Ordering
  - Numbers increase/decrease along a street
- Parity
  - Numbers on a given side of a street are odd/even

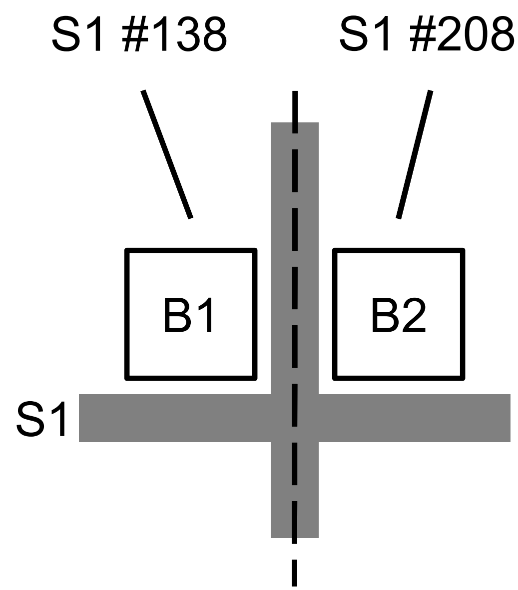


Additional information

Landmarks

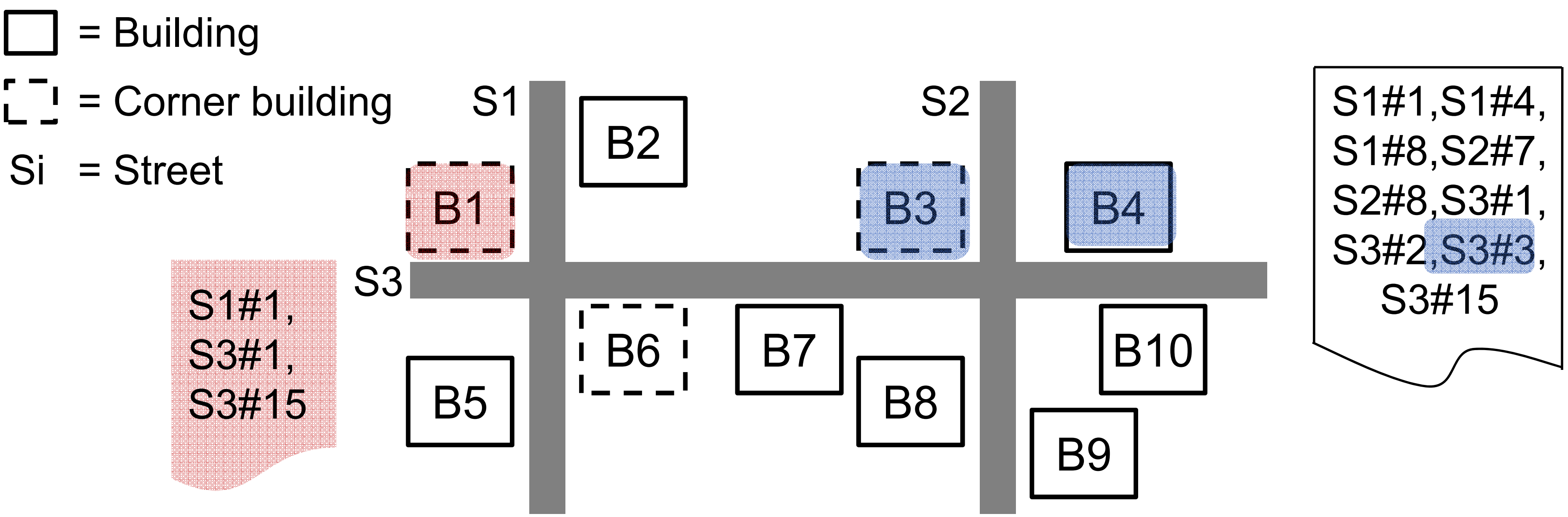


Gridlines



Query

1. Given an address, what buildings could it be?
2. Given a building, what addresses could it have?

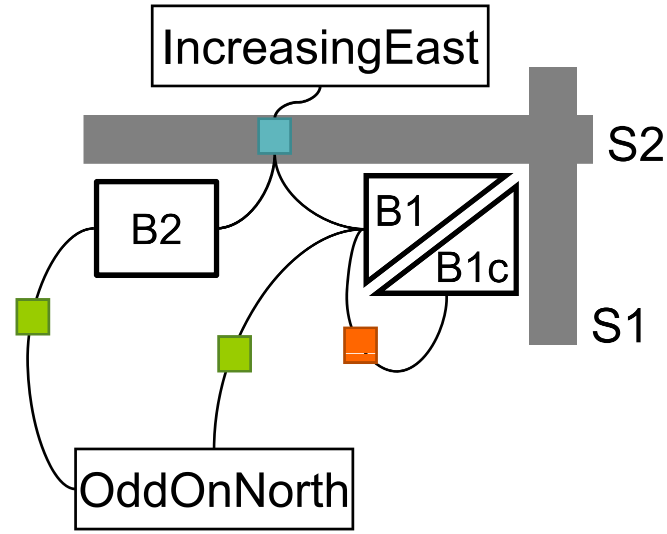


Outline

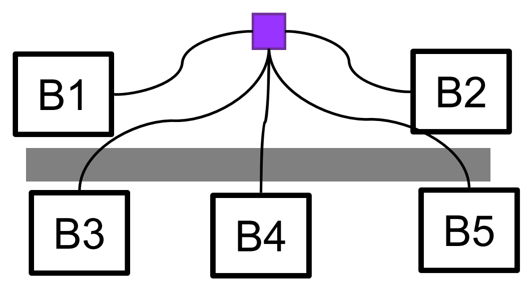
- Background
- **BID model & custom solver**
- Reformulation techniques
- Conclusions & future work

CSP model

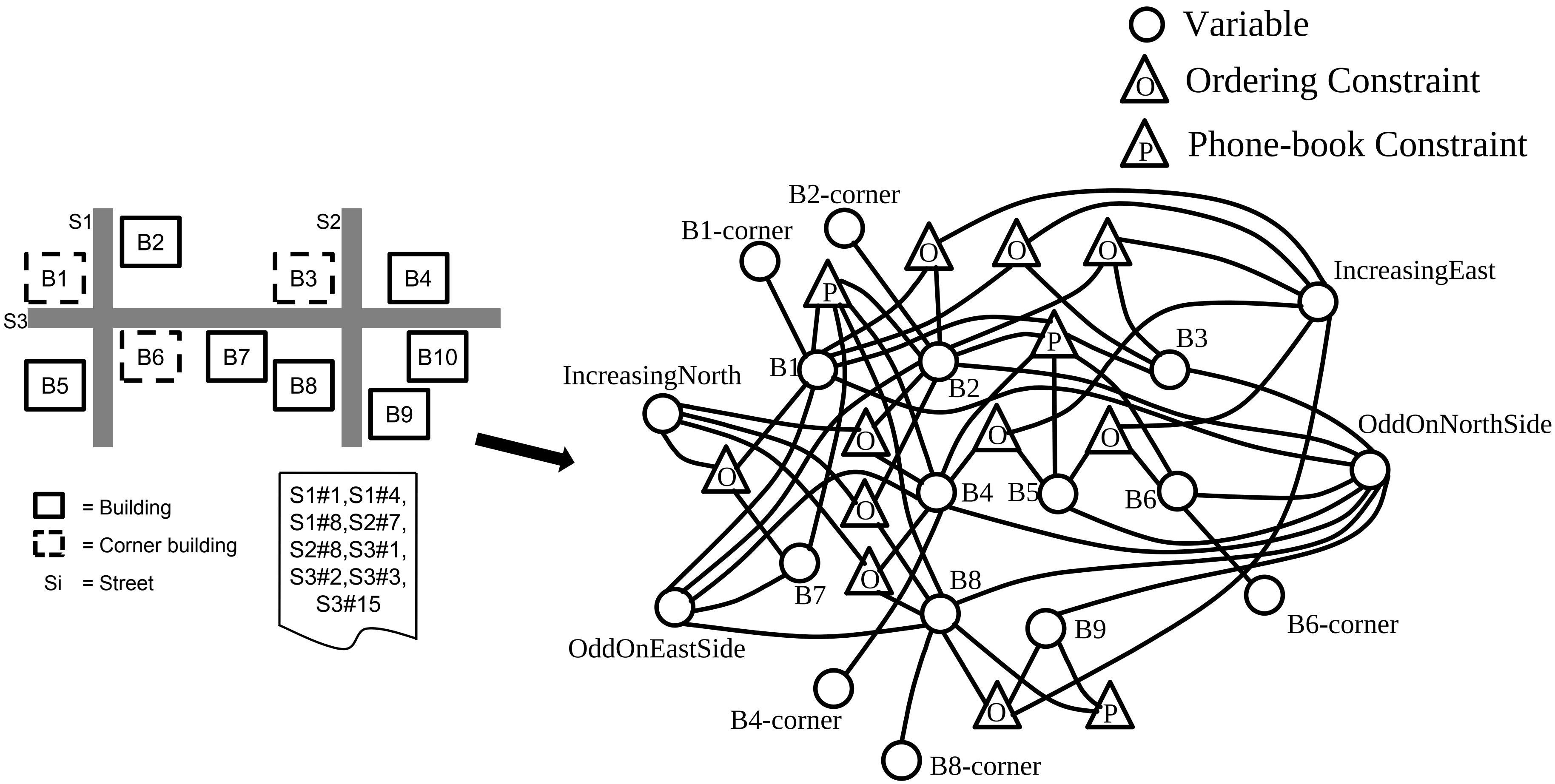
- Parity constraints
- Ordering constraints
- Corner constraints



- Phone-book constraints
- Optional: grid constraints



Example constraint network



Features of new model & solver

Improvement over previous work: [Michalowski +, 05]

- Model
  - Reduces number of variables and constraints arity
  - Reflects topology: Constraints can be declared locally & in restricted 'contexts,' important feature for Michalowski's work
- Solver
  - Exploits structure of problem (backdoor variables)
  - Implements domains as (possibly infinite) intervals
  - *Incorporates all reformulations (to be introduced)*

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  - **Query reformulation**
  - AllDiff-Atmost & domain reformulation
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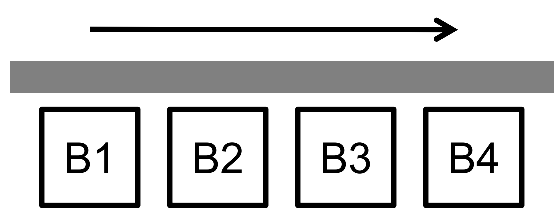
Query in the BID

- Problem: BID instances have many solutions



Phone book: {4,8}

B1	B2	B3	B4
2	4	6	8
2	4	8	10
2	4	8	12
4	8	10	12
4	6	8	10
4	6	8	12

We **only** need to know which values (address) appear in ***at least one*** solution for a variable (building)

Query reformulation

Original BID

Query:
Find **all** solutions,
Collect values for variables

Query reformulation →

Reformulated BID

Query:
For each variable-value pair,
determine **satisfiability**

Original query	Reformulated query
Single enumeration problem	Many satisfiability problems
All solutions	Per-variable solution
Exhaustive search	One path
Impractical when there are many solutions	Costly when there are few solutions

Evaluations: real-world data from El Segundo

[Shewale]

Case study	Phone book	Number of…		
	Completeness	Buildings	Corner buildings	Blocks
NSeg125-c NSeg125-i	100.0% 45.6%	125	17	4
NSeg206-c NSeg206-I	100.0% 50.5%	206	28	7
SSeg131-c SSeg131-i	100.0% 60.3%	131	36	8
SSeg178-c SSeg178-i	100.0% 65.6%	178	46	12
Previous work did not scale up beyond		34	7	1

Evaluation: query reformulation

Incomplete phone book → many solutions → better performance

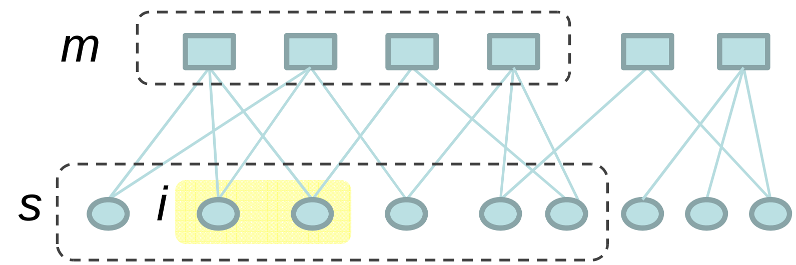
Case study	Original query	New query [s]
NSeg125-i	>1 week	744.7
NSeg206-i	>1 week	14,818.9
SSeg131-i	>1 week	66,901.1
SSeg178-i	>1 week	119,002.4

Complete phone book → few solutions → worse performance

Case study	Original query [s]	New query [s]
NSeg125-c	1.5	139.2
NSeg206-c	20.2	4,971.2
SSeg131-c	1123.4	38,618.4
SSeg178-c	3291.2	117,279.1

Generalizing query reformulation

- Relational (i,m)-consistency, algorithm R(i,m)C
  - For every m constraints
    - Compute **all solutions** of length s
    - To generate tuples of length i
  - Space: $O(d^s)$



- Query reformulation for Relational (i,m)-consistency
  - For each combination of values for i variables
    - Try to extend to **one** solution of length s
  - Space: $O(\binom{s}{i} d^i)$, $i < s$
- Reformulated BID query is R(1,$|\mathcal{C}|$)C

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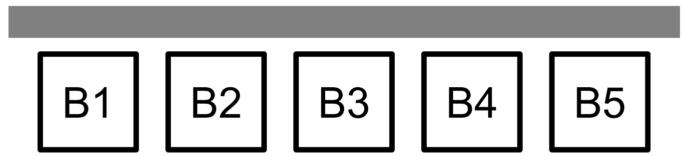
AllDiff-Atmost in the BID

Even side



Phone book: {12,48}

Original domain = {2, 4, …, 998, 1000}

12	**48**	30	32	34
12	14	16	38	**48**
10	**12**	14	20	**48**
2	4	6	**12**	**48**
…	…	**12**	**48**	**…**

- Can use at most
  - **3** addresses in [2,12) AllDiff-Atmost({B1,B2,..,B5},3,[2,12))
  - **3** addresses in (12,48) AllDiff-Atmost({B1,B2,..,B5},3,(12,48))
  - **3** addresses in (48,1000] AllDiff-Atmost({B1,B2,..,B5},3,(48,1000))

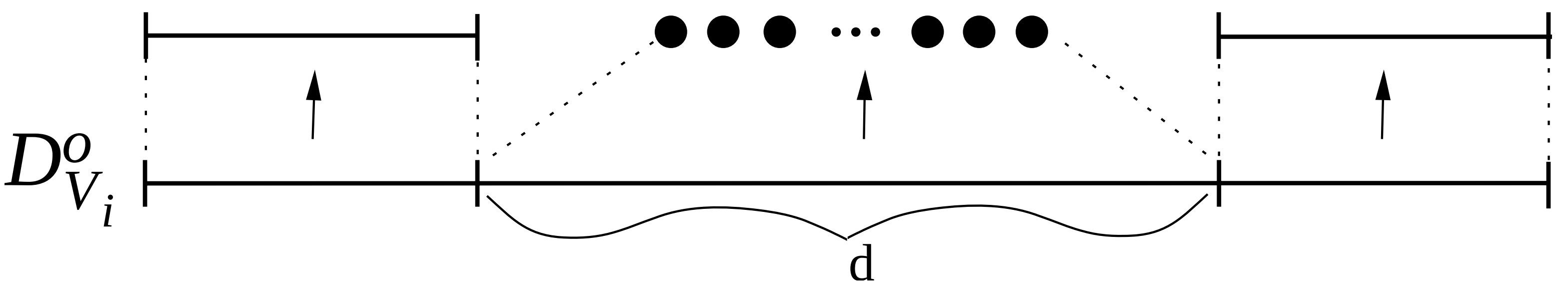
Reformulated domain { s_1, s_2, s_3, 12, s_4, s_5, s_6, 48, s_7, s_8, s_9 }

Original domain { 2, 4, …, 10, 12, 14, …, 46, 48, 30, …, 998, 1000 }

AllDiff-Atmost reformulation

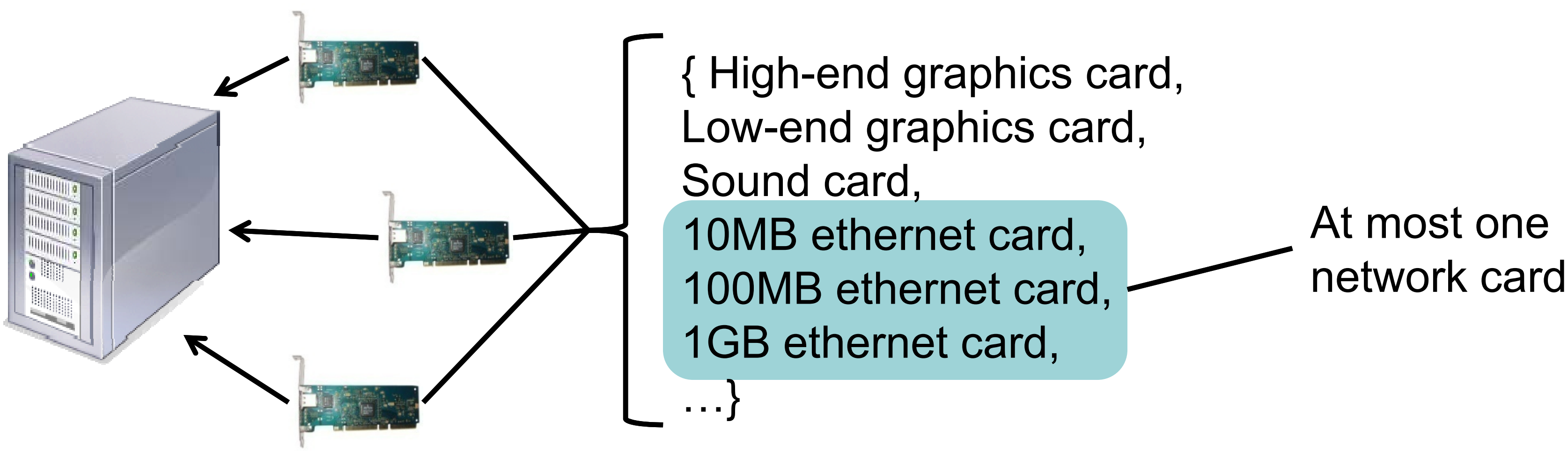
- Given AllDiff-Atmost($\mathcal{A}$,*k*,*d*)
  - The variables in $\mathcal{A}$ can be assigned at most *k* values from the set *d*
- Replace
  - interval *d* of values (potentially infinite)
  - with *k* **symbolic values**

$$D^{ref}_{V_i} = D^{ref,l}_{V_i} \cup \{S_1, S_2, \ldots, S_k\} \cup D^{ref,r}_{V_i}$$



AllDiff-Atmost constraint

- AllDiff-Atmost($\mathcal{A}$,*k*,*d*)
  - The variables in $\mathcal{A}$ can be assigned at most *k* values from the set *d*



Three expansion slots

Evaluation: domain reformulation

- Reduced domain size → improved search performance

Case study	Phone-book completeness	Average domain size		Runtime [s]	
		Original	Reformulated	Original	Reformulated
NSeg125-i	45.6%	1103.1	236.1	2943.7	744.7
NSeg206-i	50.5%	1102.0	438.8	14,818.9	5533.8
SSeg131-i	60.3%	792.9	192.9	67,910.1	66,901.1
SSeg178-i	65.6%	785.5	186.3	119,002.4	117,826.7

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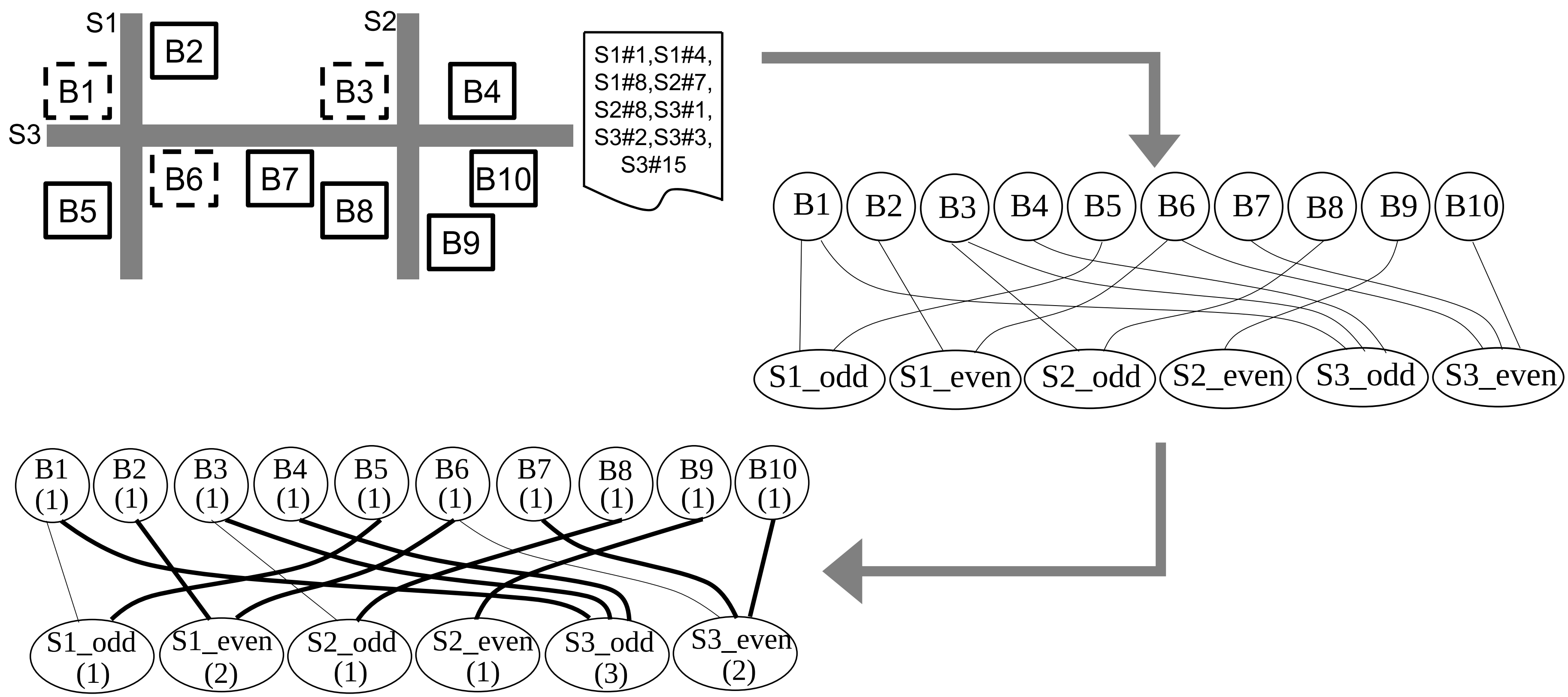
BID as a matching problem

- Assume we have no grid constraints



BID w/o grid constraints

- BID instances without grid constraints can be solved in ***polynomial time***

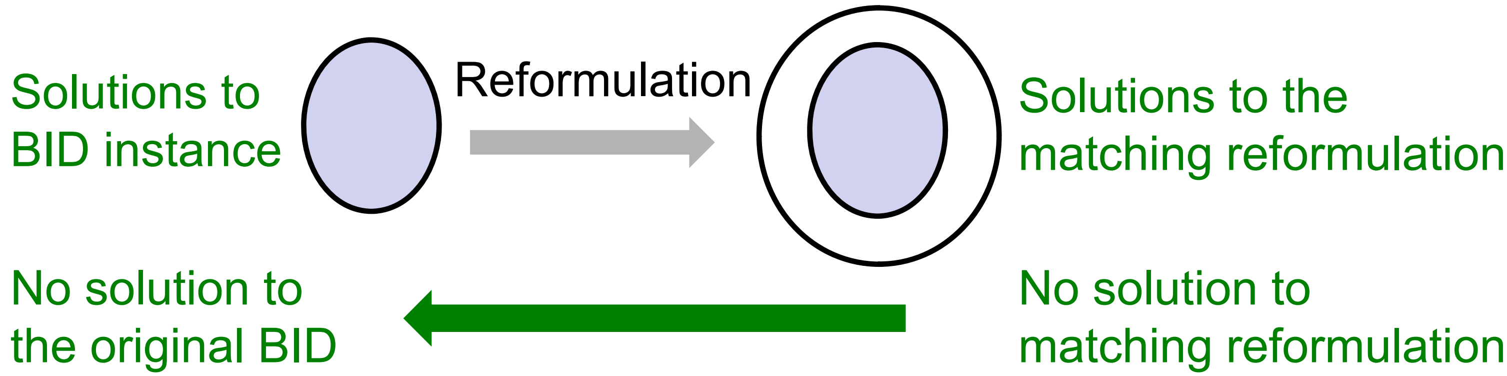
Case study	Runtime [s]	
	BT search	Matching
NSeg125-c	139.2	4.8
NSeg206-c	4971.2	16.3
SSeg131-c	38618.3	7.3
SSeg178-c	117279.1	22.5
NSeg125-i	744.7	2.5
NSeg206-i	5533.8	8.5
SSeg131-i	38618.3	7.3
SSeg178-i	117826.7	4.9

BID w/ grid constraints

1. Filter CSP [Régin, 1994]

   Remove vvps that do not appear in a maximum matching

2. Relaxed CSP: matching reformulation as a *necessary approximation* of the BID



Matching reformulation in Solver

Filter CSP.. **Preproc1**

For every vvp

Consider CSP + vvp

If relaxed CSP is solvable **Preproc2**

Find one solution using BT search

At each instantiation, filter CSP **Lookahead**

Evaluation: matching reformulation

- Generally, improves performance

Case Study	BT	Preproc2 +BT	% (from BT)	Lkhd +BT	% (from BT)	Lkhd +Preproc1&2 + BT	% (from Lkhd+BT)
NSeg125-i	1232.5	1159.1	**6.0%**	726.6	**41.0%**	701.1	**3.5%**
NSeg206-c	2277.5	614.2	**73.0%**	1559.2	**31.5%**	443.8	**71.5%**
SSeg178-i	138404.2	103244.7	**25.4%**	121492.4	**12.2%**	85185.9	**29.9%**

- Rarely, the overhead exceeds the gains

Case Study	BT	Preproc2 +BT	% (from BT)	Lkhd +BT	% (from BT)	Lkhd +Preproc1&2 + BT	% (from Lkhd+BT)
NSeg125-c	100.8	33.2	**67.1%**	**140.2**	**-39.0%**	29.8	**78.7%**
NSeg131-i	114405.9	114141.3	**0.2%**	107896.3	**5.7%**	**108646.6**	**-0.7%**

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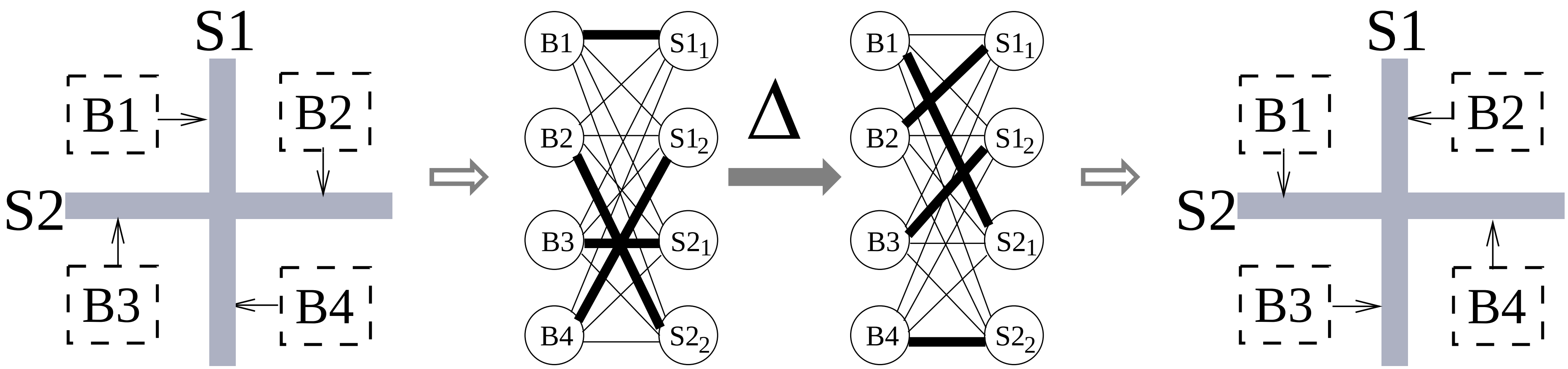
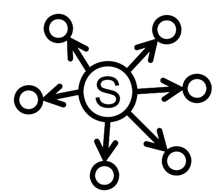
Symmetric matchings in BID



- ***All*** matchings can be produced from the symmetric difference of
  - a single matching and
  - a set of disjoint alternating cycles & paths starting @free vertex



- Some symmetric solutions do not break grid constraints
  - Ignore symmetric solutions during search
- Some do, we do not know how to use them…

Conclusions

- We proposed four reformulation techniques
- We described their usefulness for general CSPs
- We demonstrated their effectiveness on the BID

Lesson:

Reformulation is an effective approach to improve the scalability of complex combinatorial systems

Future work

- Empirically evaluate our new algorithm for relational *(i,m)-consistency*
- Exploit the symmetries we identified
- Enhance the model by incorporating new constraints [Michalowski]

Questions?